

General intrinsic theory of general large N_c QCD, SU(3) QCD, SU(2) hadron-dynamics and U(1) QED gauge field theories in general field theory and progress towards solving the nucleon spin crisis

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This paper gives general intrinsic theory of general large N_c QCD, SU(3) QCD, SU(2) hadron-dynamics and U(1) QED gauge field theories in general field theory and progress towards solving the nucleon spin crisis, i.e., presents general large N_c QCD's inner structures, gauge invariant angular momenta and new corresponding Coulomb theorem in quark-gluon field interaction systems based on general field theory, and naturally deduces the gauge invariant spin and orbital angular momentum operators of quark and gauge fields with $SU(N_c)$ gauge symmetry by Noether theorem in general field theory. In the general large N_c QCD, we discover not only the general covariant transverse and parallel conditions (namely, non-Abelian divergence and curl), but also that this general system has good intrinsic symmetry characteristics. Specially, this paper's generally decomposing gauge potential theory presents a new technique, it should play a vital role in future physics research. Therefore, this paper breakthroughs the some huge difficulties in the nucleon spin crisis and opens a door of researching on lots of strong interacting systems with different symmetric properties, which is popularly interesting, and keeps both the gauge invariance and their angular momentum commutation relations so that their theories are consistent. Especially, the achieved results here can be utilized to calculate the general QCD strong interactions and to give the precise predictions that can be exactly measured by current particle physics experiments due to their gauge invariant properties etc.

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1.Introduction

Yang and Mills, in 1954 for the first time, generalized the Abelian U(1) gauge theory to non-Abelian SU(2) gauge theory to give a strong interaction theory [1]. This theory now has been greatly developed and is viewed as the powerful and useful Yang-Mills gauge field theory. As a further development of it, SU(2) hadron-dynamics gauge field theory in high energy physics is the strong interaction theory of different states relevant to different charged electricity states.

As is well known to people, up to now, the investigations on nuclear theories have been awarded three Nobel physics prizes because of their huge impacts for physics science. It is well-known that QCD is useful for research on general strong interactions of quark-gluon systems, and large N_c QCD is currently utilized to investigate quark-gluon interactions, e.g., Ref.[2] studied Nucleon-Antinucleon Annihilation in Large N_c QCD.

The well-known nucleon spin crisis shows the key and urgent problem: how do the motions of quarks and gluons in nucleon give the contribution to the total spin of the nucleon? Various investigations show that the spin of the quarks can only give one third of the nucleon spin [3–5], therefore, the other part must be achieved from the other internal factors or motions: the spin of the gluons

and the orbital angular movements of the quarks and gluons. In order to uncover how these motions give the contribution to the total nucleon spin, the original work is generally to define the operators of fully representing these movements. Various kinds of good endeavors have been made for solving the crisis, e.g., Refs.[6–11].

We, in this paper, generalize the relevant investigations in QED [12] and isospin gauge field theory [13] in order to investigate the inner structure of the interaction systems of the quark-gluon fields under the intrinsic decomposition of color gauge fields. This kind of generalizations is nontrivial, and people will find that there must be some required additional conditions so that the new achieved theory is gauge invariant and consistent. Furthermore, people will see that all the non-Abelian consistent conditions can be impressively achieved by properly generalizing their Abelian corresponding parts that had been exactly shown in Ref.[12]. Ref.[14] had given two of the consistent conditions, while Ref.[13] further found that the other four consistent conditions show the properties of different components of the gauge field strength in SU(2) isospin gauge field theory. Because, in strong interaction systems with isospin symmetry, there still is the serious problem that gauge invariant angular momenta are still missing [15, 16], Ref.[13] solved the serious problem. This paper wants not only to generalize the solutions to the key problems, analogous to nucleon spin crisis, in QED [12] and SU(2) isospin gauge theory [13], respectively, (namely, based on the solid bases of Refs.[12, 13] at al)

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in order to make essential steps towards solving nucleon spin crisis, but also to give their consistent unification description theory.

The arrangement of this paper: Sect. 2 is $SU(N_c)$ gauge field inner structures and new Jacobi identity; Sect. 3 is general decompositions of $SU(N_c)$ gauge fields; Sect. 4 shows gauge invariant angular momenta of $SU(N_c)$ quark-gluon field interaction systems and gives a new viewpoint on the resolution of the nucleon spin crisis; Sect. 5 is discussion; Sect. 6 gives summary and conclusion.

2. $SU(N_c)$ gauge field inner structures and new Jacobi identity

For $SU(N_c)$ gauge field generators $T^a, a = 1, 2, \dots, N_c^2 - 1$; N_c may be enough large, we have

$$\begin{aligned} T^a T^b &= \frac{1}{2}(T^a T^b + T^b T^a) + \frac{1}{2}(T^a T^b - T^b T^a) \\ &= \frac{1}{2N_c} \delta^{ab} + \frac{1}{2}(d^{abe} + if^{abe})T^e, N_1 = N, N_2 = M, \end{aligned} \quad (2.1)$$

because $\{T^a, T^b\} = \frac{1}{2N_c} \delta^{ab} + \frac{1}{2}d^{abe}T^e$, $[T^a, T^b] = if^{abe}T^e$. Using Eq.(2.1), we have

$$\frac{1}{2N_c} \delta^{ab} T^c T^d = [T^a T^b - \frac{1}{2}(d^{abe} + if^{abe})T^e] T^c T^d. \quad (2.2)$$

Similar to deduction of Eq.(2.2), we get

$$\frac{1}{2N_c} \delta^{ad} T^b T^c = [T^a T^d - \frac{1}{2}(d^{ade} + if^{ade})T^e] T^b T^c, \quad (2.3)$$

Using Eqs.(2.2) and (2.3), we can have

$$\frac{1}{4N_c} (\delta^{ab} \delta^{cd} - \delta^{ad} \delta^{bc}) = \text{tr}(T^a [T^b T^c, T^d]) - \frac{1}{2}(d^{abe} + if^{abe}) \frac{1}{4} ($$

$$d^{ecd} + if^{ecd}) + \frac{1}{2}(d^{ade} + if^{ade}) \frac{1}{4} (d^{ecd} + if^{ecd}) \quad (2.4)$$

where we have used $\text{tr}(T^a T^b) = \frac{1}{2} \delta^{ab}$ and $\text{tr}(T^a T^b T^c) = \frac{1}{4} (d^{abc} + if^{abc})$ [16].

Using $[T^b T^c, T^d] = T^b [T^c, T^d] + [T^b, T^d] T^c$ and further simplifying Eq.(2.4), we deduce

$$\frac{2}{N_c} (\delta^{ab} \delta^{cd} - \delta^{ad} \delta^{bc}) + d^{abe} d^{cde} = f^{ace} f^{bde} + d^{ade} d^{bce} +$$

$$i(d^{abe} f^{cde} + 2d^{ace} f^{bde} - f^{abe} d^{ecd} + f^{ade} d^{ebc} + d^{ade} f^{ebc}) \quad (2.5)$$

Using Eq.(2.5), we achieve both a general contraction expression of structure constants of $SU(N_c)$ gauge group

$$f^{ace} f^{bde} = \frac{2}{N_c} (\delta^{ab} \delta^{cd} - \delta^{ad} \delta^{bc}) + d^{abe} d^{cde} - d^{ade} d^{bce} \quad (2.6)$$

and an equation

$$d^{abe} f^{cde} + 2d^{ace} f^{bde} - f^{abe} d^{ecd} + f^{ade} d^{ebc} + d^{ade} f^{ebc} = 0 \quad (2.7)$$

Using Jacobi identity $d^{abe} f^{cde} + d^{ace} f^{bde} + f^{ade} d^{ebc} = 0$ [16] to simplify Eq.(2.7), we deduce a new Jacobi identity of structure constants

$$d^{ace} f^{bde} - f^{abe} d^{ecd} + d^{ade} f^{ebc} = 0, \quad (2.8)$$

because Eq.(2.8) can directly be deduced by

$$\{T^a, [T^b, T^c]\} - [T^b, \{T^c, T^a\}] - \{T^c, [T^a, T^b]\} = 0 \quad (2.9)$$

Eq.(2.9) is just a new Jacobi identity, which can be proved by directly expanding Eq.(2.9). Eq.(2.8) gives a new relation between symmetric and antisymmetric structure constants.

The other more investigations on inner structures of $SU(N_c)$ gauge fields will be naturally given in following sections.

3. General decompositions of $SU(N_c)$ gauge fields

As a general generalization of the QED and the isospin cases, we investigate general decompositions of $SU(N_c)$ gauge fields by the general method of field theory. The projection operators are defined as [12, 13]

$$L_k^j = \partial^j \frac{1}{\Delta} \partial_k, T_k^j = \delta_k^j - L_k^j, (\Delta = \partial_k \partial^k) \quad (3.1)$$

and the gauge potential $A^{ia} (i = 1, 2, 3; a = 1, 2, 3, \dots, N_c^2 - 1)$ are intrinsically decomposed as

$$A_{\perp}^{ja} = T_k^j A^{ka}, A_{\parallel}^{ja} = L_k^j A^{ka}. \quad (3.2)$$

The general action for the $SU(N_c)$ quark-gluon field interaction system is [16]

$$S = \int d^4x [-\frac{1}{4} F^{a\mu\nu} F_{a\mu\nu} + \bar{\psi} (i\gamma^\mu D_\mu - m) \psi], \quad (3.3)$$

where $D_\mu = \partial_\mu - igA_\mu$ are the covariant derivative with gauge fields $A_\mu = A_\mu^a T^a$ and T^a the generator of the $SU(N_c)$ group, and ψ stands for the quark field. When D_μ acts on any vector in Lie algebra space, we have $D_\mu B = \partial_\mu B^a T^a - ig[A_\mu, B^a T^a] = (\partial_\mu B^a + gf^{abc} A_\mu^b B^c) T^a$. In $SU(N_c)$ gauge field theory, the gauge field strength is

$$F^{a\mu\nu} = \partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + gf^{abc} A^{b\mu} A^{c\nu}. \quad (3.4)$$

We can decompose the spacial part of the gauge potential A^{ak} into two components

$$A^{ak} = A_{\perp}^{ak} + A_{\parallel}^{ak}. \quad (3.5)$$

Analogous to QED [12], under a $SU(N_c)$ gauge transformation U , $A_{\perp}^k = A_{\perp}^{ak} T^a$ and $A_{\parallel}^k = A_{\parallel}^{ak} T^a$ transform as [13]

$$A_{\perp}^{'k} = U A_{\perp}^k U^\dagger, \quad (3.6)$$

$$A_{\parallel}^{'k} = U A_{\parallel}^k U^{\dagger} - \frac{i}{g} U \partial^k U^{\dagger}, \quad (3.7)$$

respectively, which make

$$A^{'k} = U A^k U^{\dagger} - \frac{i}{g} U \partial^k U^{\dagger}. \quad (3.8)$$

The expressions from Eq.(3.5) to Eq.(3.8) simplify the relevant expressions in Ref.[13].

Using Eqs.(3.2) and (3.5), the general action (3.3) can be rewritten as

$$S = \int d^4x \left[-\frac{1}{2} (\partial^k A^{a0} \partial_k A_0^a + \partial^0 A_{\parallel}^{ak} \partial_0 A_{\parallel k}^a + \partial^0 A_{\perp}^{ak} \partial_0 A_{\perp k}^a) \right. \\ \left. + \partial^k A^{a0} \partial_0 A_{\parallel k}^a + \partial^k A^{a0} \partial_0 A_{\perp k}^a + \partial^0 A_{\parallel}^{ak} \partial_0 A_{\perp k}^a - \partial^k A^{a0} (A_{\parallel k}^b A_0^c + A_{\perp k}^b A_0^c) g f^{abc} \right. \\ \left. + \partial^0 A_{\parallel}^{ak} g f^{abc} (A_{\parallel k}^b A_0^c + A_{\perp k}^b A_0^c) + \partial^0 A_{\perp}^{ak} g f^{abc} (A_{\parallel k}^b A_0^c + A_{\perp k}^b A_0^c) - \frac{1}{2} g f^{abc} (A_{\parallel}^{bk} A^{c0} + A_{\perp}^{bk} A^{c0}) \right] \quad (3.9)$$

$$A_{\parallel k}^{'b} A_0^{'c} + A_{\perp k}^{'b} A_0^{'c}) g f^{ab'c'} - \frac{F^{ajk} F_{jk}^a}{4} + \bar{\psi} (i \gamma^{\mu} D_{\mu} - m) \psi] \quad (3.9)$$

In order to cancel the non-independent terms in Eq.(3.9), we need to use the basic consistent conditions in $SU(N_c)$ gauge theory to simplify Eq.(3.9).

Because the nonlinearity of the non-Abelian $SU(N_c)$ gauge field system, the non-Abelian system is much more complex than the Abelian one. We now generalize the two well known Abelian gauge potential (or field) consistency conditions in QED [12]

$$\nabla \cdot \vec{A}_{\perp} = 0, \quad (3.10)$$

$$\nabla \times \vec{A}_{\parallel} = 0, \quad (3.11)$$

respectively to non-Abelian $SU(N_c)$ corresponding expressions

$$\partial_k A_{\perp}^{ak} + g f^{abc} A_{\parallel k}^b A_{\perp}^{ck} = 0. \quad (3.12)$$

$$(D_{\parallel}^j D_{\parallel}^k - D_{\parallel}^k D_{\parallel}^j)^a = 0, i.e. \partial^j A_{\parallel}^{ak} - \partial^k A_{\parallel}^{aj} + g f^{abc} A_{\parallel}^{bj} A_{\parallel}^{ck} = 0 \quad (3.13)$$

One can easily prove that Eqs.(3.12) and (3.13) are $SU(N_c)$ gauge covariant, and they are similar to the isospin relevant expressions in Ref.[13] except the structure constants and superscripts' taking values of $SU(N_c)$ gauge group and being related to gauge fields [16].

Using Eq.(3.12) we can have

$$\int d^3x \partial^k A^{a0} \partial_0 A_{\perp k}^a - \int d^3x g f^{abc} \partial_0 A_{\parallel k}^a A_{\perp}^{bk} A^{c0}$$

$$+ \int d^3x g f^{abc} \partial_0 A_{\perp}^{ak} A_{\parallel k}^b A^{c0} = 0 \quad (3.14)$$

where the divergence term has been neglected.

We further utilize the consistent conditions (whose physics meanings will be explained latter)

$$\int d^3x \partial^0 A_{\parallel}^{ak} (-\partial_0 A_{\perp k}^a + g f^{abc} A_{\perp k}^b A^{c0}) = 0, \quad (3.15)$$

$$g f^{abc} (\partial_0 A_{\parallel}^{ak} A_{\perp k}^b A^{c0} - \partial^k A^{a0} A_{\perp k}^b A_0^c) \\ - \frac{g^2}{N_c} (A_{\parallel}^{bk} A^{c0} A_{\perp k}^b A_0^c - A_{\parallel}^{bk} A^{c0} A_{\perp k}^c A_0^b)$$

$$- g^2 (d^{abb'} d^{acc'} - d^{abc'} d^{acb'}) A_{\parallel}^{bk} A^{c0} A_{\perp k}^{'b} A_0^{'c} = 0, \quad (3.16)$$

and substitute the contraction expression (2.6) of structure constants of $SU(N_c)$ gauge group into Eq.(3.9), we achieve

$$S = \int d^4x \left\{ -\frac{1}{2} (\partial^k A^{a0} \partial_k A_0^a + \partial^0 A_{\parallel}^{ak} \partial_0 A_{\parallel k}^a + \partial^0 A_{\perp}^{ak} \partial_0 A_{\perp k}^a) \right. \\ \left. + \partial^k A^{a0} \partial_0 A_{\parallel k}^a - \partial^k A^{a0} (g f^{abc} A_{\parallel k}^b A_0^c + g f^{abc} A_{\perp k}^b A_0^c) + \partial^0 A_{\parallel}^{ak} g f^{abc} A_{\parallel k}^b A_0^c + \partial^0 A_{\perp}^{ak} g f^{abc} A_{\perp k}^b A_0^c - \frac{g^2}{N_c} (A_{\parallel}^{bk} A_{\parallel k}^b A^{c0} A_0^c \right. \\ \left. - A_{\parallel}^{bk} A_0^b A_{\parallel k}^c A^{c0} + A_{\perp}^{bk} A_{\perp k}^b A^{c0} A_0^c - A_{\perp}^{bk} A_0^b A_{\perp k}^c A^{c0}) - \frac{1}{2} g^2 (d^{abb'} d^{acc'} - d^{abc'} d^{acb'}) (A_{\parallel}^{bk} A^{c0} A_{\parallel k}^{'b} A_0^{'c} + \right. \\ \left. A_{\perp}^{bk} A^{c0} A_{\perp k}^{'b} A_0^{'c}) - \frac{1}{4} F^{ajk} F_{jk}^a + \bar{\psi} (i \gamma^{\mu} D_{\mu} - m) \psi \right\} \quad (3.17)$$

One can see that there is no the cross terms between parallel and vertical gauge potentials in Eq.(3.17), which just shows that the general system has very good symmetry properties, i.e., we discover that this general system has very good intrinsic symmetry characteristics.

Therefore, we can calculate the corresponding canonical momenta conjugate to the two components of the gauge potential, respectively,

$$\pi_{\parallel}^{ak} = \frac{\delta L}{\delta (\partial_0 A_{\parallel k}^a)} = -\partial^0 A_{\parallel}^{ak} + \partial^k A^{a0} + g f^{abc} A_{\parallel}^{bk} A^{c0} \quad (3.18)$$

$$\pi_{\perp}^{ak} = \frac{\delta L}{\delta (\partial_0 A_{\perp k}^a)} = -\partial^0 A_{\perp}^{ak} + g f^{abc} A_{\perp}^{bk} A^{c0} \quad (3.19)$$

Eqs.(3.18) and (3.19) are the direct generalization of the isospin relevant expressions in Ref.[13] with the structure constants and superscripts' taking values of $SU(N_c)$

gauge group and are related to gauge fields [14]. It is apparent that Eqs.(3.18) and (3.19) are both gauge covariant and their summation is exactly the conventional π^{ak} , which show that the above investigations are consistent.

Consequently, we need further to generalize, in QED respectively, the well-known Abelian gauge field strength consistent conditions [14]

$$\nabla \cdot \vec{E}_\perp = 0, \quad (3.20)$$

$$\nabla \times \vec{E}_\parallel = 0, \quad (3.21)$$

$$\nabla \cdot \vec{E}_\parallel = \rho_e, \quad (3.22)$$

to non-Abelian $SU(N_c)$ corresponding expressions

$$\partial_k \pi_\perp^{ak} + g f^{abc} A_{\parallel k}^b \pi_\perp^{ck} = 0. \quad (3.23)$$

$$D^j \pi_\parallel^{ak} - D^k \pi_\parallel^{aj} = 0, \quad (3.24)$$

$$\partial_k \pi_\parallel^{ak} + g f^{abc} A_{\parallel k}^b \pi_\parallel^{ck} = \rho^a = g \psi^\dagger T^a \psi, \quad (3.25)$$

where ρ^a is the non-Abelian charge density.

4. Gauge invariant angular momenta of $SU(N_c)$ quark-gluon field interaction systems and a new viewpoint on the resolution of the nucleon spin crisis

Using Noether theorem, the action (3.17) results in the angular momentum of the $SU(N_c)$ quark-gluon field interaction system as follows

$$\begin{aligned} \vec{J}_1 = & \int d^3x \psi^\dagger \frac{1}{2} \vec{\Sigma} \psi + \int d^3x \psi^\dagger \vec{x} \times \frac{1}{i} \nabla \psi + \int d^3x \pi_\perp^a \times \vec{A}_\perp^a + \\ & \int d^3x \pi_\parallel^a \times \vec{A}_\parallel^a + \int d^3x \pi_{\perp k}^a \vec{x} \times \nabla A_\perp^{ak} + \int d^3x \pi_{\parallel k}^a \vec{x} \times \nabla A_\parallel^{ak}. \end{aligned} \quad (4.1)$$

Actually, Eq.(4.1) is a straightforward generalization that in Ref.[13].

In order to simplify Eq.(4.1), using Eq.(3.13) we have

$$\begin{aligned} \nabla \cdot [\pi_\parallel^a (\vec{A}_\parallel^a \times \vec{x})] = & -\vec{\pi}_\parallel^a \times \vec{A}_\parallel^a - \pi_{\parallel i}^a \vec{x} \times \nabla A_\parallel^{ai} \\ & - (\partial_k \pi_\parallel^{ak} + g f^{abc} A_{\parallel k}^b \pi_\parallel^{ck}) (\vec{x} \times \vec{A}_\parallel^a). \end{aligned} \quad (4.2)$$

Utilizing the divergence consistency condition for parallel components (3.25) of gauge field strength and the identity (4.2), we can simplify Eq.(4.1) as

$$\begin{aligned} \vec{J}_2 = & \int d^3x \psi^\dagger \frac{1}{2} \vec{\Sigma} \psi + \int d^3x \psi^\dagger \vec{x} \times \frac{1}{i} \vec{D}_\parallel \psi \\ & + \int d^3x \pi_\perp^a \times \vec{A}_\perp^a + \int d^3x \pi_{\perp k}^a \vec{x} \times \nabla A_\perp^{ak} \end{aligned}$$

$$= \vec{S}^q + \vec{L}_2^q + \vec{S}_2^g + \vec{L}_2^g, \quad (4.3)$$

in which, because of Eq.(3.7), $\vec{D}_\parallel = \nabla - ig \vec{A}_\parallel$ is the new covariant derivative. \vec{S}^q , \vec{L}_2^q , \vec{S}_2^g and \vec{L}_2^g are the spin and orbital angular momenta of the quark field and gauge field, respectively. Eq.(4.3) is just a direct generalization of the relevant expression of $SU(2)$ isospin symmetry theory in Ref.[13] because of their similar mathematical structure.

Due to the properties of A_\parallel^{ak} shown in Eqs.(3.7) and (3.13), \vec{L}_2^g satisfies the commutation law $\vec{L}_2^g \times \vec{L}_2^g = i \vec{L}_2^g$ and is gauge invariant [6]. The gauge invariance of \vec{S}^q and \vec{S}_2^g is obvious; furthermore, A_\perp^{ak} is gauge covariant in Eq.(4.3), which means that \vec{L}_2^g is gauge invariant.

Under the general Lorentz transformation of the fundamental fields, Noether's theorem requires the invariance of the Lagrangian and Hamiltonian densities of the general system [17, 18], e.g., this system is invariant under the Lorentz transformations of A^μ 's all components, although Lorentz transformation of A_\perp^μ is complex, the relevant complex terms of the transformations of all their components can be canceled each other in the whole system according to the symmetric invariance property of this system, which is the general rule, see Refs.[17, 18]. Furthermore, the frame-dependence issue has important physics meaning, Refs.[9–11] have given the very good investigations on the issue etc.

Next, we will derive some useful relations from the consistency conditions for field strength. Making gauge transformation of Eq.(4.3)'s the last term, we have

$$\begin{aligned} tr(\pi'_{\perp k} \vec{x} \times \nabla A_{\perp}^{k'}) = & tr(\pi_{\perp k} \vec{x} \times \nabla A_{\perp}^k) \\ & + tr(x \times U^+ \nabla U [A_{\perp k}, \pi_{\perp}^k]) = 0, \end{aligned} \quad (4.4)$$

thus gauge invariance of the last term of Eq.(4.3) demands

$$f^{abc} A_{\perp k}^b \pi_{\perp}^{ck} = 0, \quad (4.5)$$

consequently we naturally deduce a new consistent condition [6].

Then, with the well known Coulomb law in QCD [16]

$$\partial_k \pi^{ak} + g f^{abc} A_k^b \pi^{ck} = \rho^a, \quad (4.6)$$

and in terms of Eqs.(3.23), (3.25) and (4.6), we get

$$g f^{abc} A_{\perp k}^b \pi^{ck} = 0. \quad (4.7)$$

Using the consistent condition (4.5) of gauge invariance of the last term of Eq.(4.3), we can simplify Eq.(4.7) as

$$f^{abc} A_{\perp k}^b \pi_{\parallel}^{ck} = 0, \quad (4.8)$$

which can be used to cancel some extra freedoms of this system.

Furthermore, in order to illustrate the meanings of the two consistent conditions (3.15) and (3.16) in simplifying the original action, substituting Eq.(3.19) into Eq.(3.15) and using Eq.(3.18), we show that Eq.(3.15) is just the orthogonal relation between the two components of the canonical momentum

$$\int d^3x \pi_{\parallel}^{ak} \pi_{\perp k}^a = 0. \quad (4.9)$$

which is just a generalization of Abelian orthogonal relation $\int d^3x E_{\perp}^k E_{\parallel k} = 0$ (in the sense of the whole space) to a non-Abelian case.

Then, substituting Eq.(3.19) into Eq.(3.23) and using Eqs.(2.6) and (3.12), we can prove that

$$g f^{abc} \partial^0 A_{\parallel k}^b A_{\perp}^{ck} + g f^{abc} A_{\perp k}^{bk} \partial_k A^{c0} - g^2 \frac{2}{N_c} (A_{\parallel k}^b A_{\perp}^{bk} A^{a0} + A_{\parallel k}^a A_{\perp}^{ck} A^{c0}) + g^2 (d^{abc} d^{b'a'c} - d^{ab'c} d^{ba'c}) A_{\parallel k}^b A_{\perp}^{a'k} A^{b'0} = 0 \quad (4.10)$$

Multiplying its right side by A_0^a and rearranging the $SU(N_c)$ group indices, Eq.(4.10) results in Eq.(3.16).

Consequently, all the assumptions being made here are consistent, namely, Eqs.(3.15), (3.16) and (4.5) can be given out from Eqs.(3.12), (3.23), (3.25) and (4.9), which are just directly generalized from the Abelian conditions to non-Abelian cases in order to keep their gauge covariant way and their consistent properties.

Furthermore, we will show the relation between our result and the previous ones. To do this, we need to give another expression of angular momenta J_2 . Using Eqs.(3.12), (3.13) and (4.8), we achieve the surface term

$$\begin{aligned} \nabla \cdot [A_{\perp}^a (\pi_{\parallel}^a \times \vec{x})] &= \vec{\pi}_{\parallel}^a \times \vec{A}_{\perp}^a + \pi_{\parallel}^{ak} \vec{x} \times \nabla A_{\perp k}^a \\ &+ A_{\perp k}^a \varepsilon^{lmn} x_n \vec{e}_l (D_{\parallel}^k \pi_{\parallel m}^a - D_{\parallel m} \pi_{\parallel}^{ak}), \end{aligned} \quad (4.11)$$

where \vec{e}_l is the spatial unit vector and $D_{\parallel}^k \pi_{\parallel m}^a = \partial^k \pi_{\parallel m}^a + g f^{abc} A_{\parallel}^{bk} \pi_{\parallel m}^c$, because there is generalizing the Abelian Eq.(3.21) to non-Abelian gauge expression (3.24). Adding Eqs.(3.17) and (3.18) to Eq.(4.3), we deduce Chen et al's useful separation of the angular momentum [6]

$$\begin{aligned} \vec{J}_3 &= \int d^3x \psi^\dagger \frac{1}{2} \vec{\Sigma} \psi + \int d^3x \psi^\dagger \vec{x} \times \frac{1}{i} \vec{D}_{\parallel} \psi \\ &+ \int d^3x \vec{\pi}^a \times \vec{A}_{\perp}^a + \int d^3x \pi_k^a \vec{x} \times \nabla A_{\perp}^{ak} \end{aligned}$$

$$= \vec{S}^a + \vec{L}_3^a + \vec{S}_3^a + \vec{L}_3^g. \quad (4.12)$$

Therefore, in general field theory, we prove that Eq. (4.12) can strictly and naturally be given out from the action (3.17) by Noether theorem. Analogous to the derivation of Eq. (4.5), the gauge invariance of \vec{L}_3^g demands the condition (4.7) (or Eqs. (4.6) and (3.23)), which shows a key role in transforming Eq. (4.12) to the conventional form [6]

$$\begin{aligned} \vec{J}_4 &= \int d^3x \psi^\dagger \frac{1}{2} \vec{\Sigma} \psi + \int d^3x \psi^\dagger \vec{x} \times \frac{1}{i} \vec{\nabla} \psi \\ &+ \int d^3x \vec{\pi}^a \times \vec{A}^a + \int d^3x \pi_k^a \vec{x} \times \nabla A^{ak} \\ &= \vec{S}^a + \vec{L}_4^a + \vec{S}_4^a + \vec{L}_4^g, \end{aligned} \quad (4.13)$$

Consequently, under certain conditions, we can see that \vec{J}_2 , \vec{J}_3 and \vec{J}_4 present the same value of the total angular momenta. However, the last three terms of \vec{J}_4 are apparently all gauge dependent, thus \vec{J}_4 is not properly defined. In comparison, both \vec{J}_2 and \vec{J}_3 are more physical. Furthermore, \vec{J}_2 is the simplest and the most rational one for the sake of simplicity, because \vec{J}_2 eliminates the terms that don't contribute to the total angular momentum for large N_c gauge field theory, as Eq. (4.11) shows.

5. Discussion

The nonlinear properties of interaction systems of the non-Abelian quark-gluon fields produce considerable complex properties, and the consistency and gauge invariance of the quark-gluon interaction system theory naturally require to establish the six equations: (3.12), (3.13), (3.23-3.25) and (4.9), which are directly uniformly generalized from their Abelian expressions to their non-Abelian corresponding expressions according to their gauge covariance and consistent properties, thus they can be directly simplified as the Abelian corresponding expressions when their structure constants f^{abc} are taken as zero. In order to give very obviously direct physics meanings of the six equations we rewrite them as the direct compact expressions

$$\vec{D}_{\parallel} \cdot \vec{A}_{\perp}^a = 0, \quad (5.1)$$

$$(\vec{D}_{\parallel} \times \vec{D}_{\parallel})^a = 0, \quad (5.2)$$

$$\vec{D}_{\parallel} \cdot \vec{\pi}_{\perp}^a = 0, \quad (5.3)$$

$$\vec{D}_{\parallel} \times \vec{\pi}_{\parallel}^a = 0, \quad (5.4)$$

$$\vec{D}_{\parallel} \cdot \vec{\pi}_{\parallel}^a = \rho^a, \quad (5.5)$$

$$\int d^3x \vec{\pi}_{\parallel}^a \cdot \vec{\pi}_{\perp}^a = 0. \quad (5.6)$$

Given an arbitrary gauge field configuration $A_{\nu}(x)$, one can really find an explicit solution \vec{A}_{\perp} ($\vec{A}_{\parallel} = \vec{A} - \vec{A}_{\perp}$ is not independent), which satisfies all the six consistent conditions (5.1-5.6), which guarantee the consistency of the achieved theory in this paper. Otherwise any new theory cannot return to the well-known Abelian theory [18], which leads to no consistent theory.

Generalization from Abelian Eqs.(3.10) and (3.11) to non-Abelian Eq.(3.12) and $\partial^j A_{\parallel}^{ak} - \partial^k A_{\parallel}^{aj} + g f^{abc} A_{\parallel}^{bj} A_{\parallel}^{ck} = 0$ and their important physics meanings, respectively, are given by Wang et al [14]. This paper also gives the better and more direct expression (3.13) or (5.2) for non-Abelian quark-gluon interaction system in general case than that in Ref.[13]. Consequently, the compact Eqs.(5.2-5.6) with direct physics meanings for a general large N_c QCD system in a general field theory are discovered in this paper. The non-Abelian Eqs.(3.23) and (3.24), generalized from the Abelian Eqs. (3.20) and (3.21), are the corresponding non-Abelian divergence and curl generalizations, respectively (namely, the non-Abelian transverse and parallel conditions); Eq. (5.5) is generalized from Eq.(3.22) and is the new $SU(N_c)$ Coulomb law; the orthogonal relation (5.6) is the generalized expression for the Abelian $\int d^3x \vec{E}_{\parallel}^a \cdot \vec{E}_{\perp}^a = 0$. Furthermore, we show also that Eq.(4.7) with a similar expression, imposed by Chen et al [6], which is key to maintain \vec{L}_3^g gauge invariant, is just a direct result of Eqs.(3.23), (3.25) and (4.6). Under the six consistent conditions, we naturally achieve the simplest gauge invariant expression (4.3) of the total angular momentum of a general QCD system with a general N_c —the general quark-gluon field interaction system by Noether theorem in general field theory.

For all the above general investigations with a general N_c , we get a general theory of general large N_c QCD's inner structure and its gauge invariant angular momentum and the corresponding new general Coulomb theorem etc so that we give the new viewpoint on the resolution of the nucleon spin crisis relative to the large N_c in a general situation. Specially, this theory is also suitable for investigating lattice QCD. For instance, one can use this method to discuss the theory of the lattice QCD with large N_c [16].

When the $N_c = 3$, it follows from Eq.(2.6) that

$$f^{ace} f^{bde} = \frac{2}{3}(\delta^{ab} \delta^{cd} - \delta^{ad} \delta^{bc}) + d^{abe} d^{cde} - d^{ade} d^{bce} \quad (5.7)$$

i.e., the current QCD [16] is considered. Thus substituting Eq.(5.7) into all its relevant expressions in all the above investigations, then we get a general theory of QCD's inner structure and its gauge invariant angular momentum as well as the corresponding new Coulomb theorem etc so that we give out the new viewpoint on

the resolution of the nucleon spin crisis in a general field theory. The gauge invariance and angular momenta commutation relation are not satisfied simultaneously in the frame of general field theory of QCD in the past, however, this paper, for the first time, makes these satisfied in the same time in a general field theory, without artificial choice.

When the $N_c = 2$, Eq.(2.6) is reduced as

$$f^{ace} f^{bde} = \delta^{ab} \delta^{cd} - \delta^{ad} \delta^{bc}, \quad (5.8)$$

because d^{abe} can only be equivalently taken as d^{123} here, then replacing the quark-gluon interaction with the fermi-gauge interaction, the above theory is simplified as the Ref.[13]'s investigations and which are the exact same as Ref.[13] results, i.e., the current hadron-dynamics gauge field theory about strong interaction with different charged electricity systems [13] is achieved.

When the $N_c = 1$, people usually take U(1) gauge symmetry, i.e., the Abelian gauge field is considered, its structure constant is zero, thus replacing the quark-gluon interaction with the fermi-gauge interaction, the above theory is simplified as the Ref.[12]'s investigations and which are the exact same as Ref.[12]'s results.

6. Summary and conclusion

Consequently, the above theory not only gives the general intrinsic theory of general large N_c QCD, SU(3) QCD, SU(2) hadron-dynamics and U(1) QED gauge field theories in general field theory, but also and specially meaningfully makes essential steps towards solving nucleon spin crisis in the general field theory, i.e., presents progress towards solving the nucleon spin crisis. The discoveries of the inner-structure similarity among the above theories, their gauge invariant angular momenta and the corresponding new Coulomb theorem etc of the non-Abelian fermi-gauge field interaction systems with a general large N_c vividly uncover the consistency and gauge invariance of the gauge field theory and will further deepen our understanding on some basic questions in physics.

Because of the universality of Eqs. (5.1)-(5.6), they can be applied to treat the inner structure and other aspects of any fermi-gauge interaction system, including SU(N) gauge field theory and different gauge theories for the fundamental interactions of the universe. Their aspects like Euler-Lagrange equations, corresponding conservation quantities and relevant dynamics etc. need to be revised and improved by using the new general theory of the decomposed non-Abelian gauge fields. Due to the length limit of the paper, lots of applications of this paper's theory will be written in our following works.

This paper not only presents the simplest expression (4.3) but also gives the current expression (4.12), and discovers their relation between Eqs.(4.3) and (4.12).

This paper decomposes the gluon fields into transverse and longitudinal parts in general field theory, which are then used to obtain a general expression for the total angular momentum J of a general quark-gluon system in

general field theory. This expression divides J into quark spins and gluon spins and their orbital angular momentum parts in general field theory, with gauge invariant expressions for each of the contributions.

This paper is one in a long history of attempting to decompose the nucleon's spin into various pieces which together must sum to one half. Such an effort is really of general interest because if the individual parts are either measurable experimentally or directly computable in the physics. This paper just gives the theory as how the individual parts of the spin in the decomposition can be obtained theoretically and experimentally because all the achieved quantities are exactly calculated and gauge invariant, which then results in that they are measurable, because the quantities without gauge invariance cannot be exactly measured [6].

This paper's generally decomposing gauge potential theory also presents a new technique, or methodology, it should play a vital role in future physics research, and make apparent its immediate consequences for physicists, because the generally decomposing gauge potential theory in general field theory can be directly and effectively applied to arbitrary $SU(N)$, $SO(N)$ etc gauge interaction systems with different Fermi and the other matter fields.

Therefore, this paper makes a breakthrough progress in overcoming the key huge difficulty of nucleon spin crisis,

i.e. succeeding in figuring out a natural way to construct, for the first time, the consistent angular momentum operators with both their commutation relation and gauge invariance satisfied in general field theory, and further a new door is eventually opened to investigate lots of strong interacting systems with different symmetric properties, which is very important in understanding the nature and thus popularly interesting, and their theories are consistent. These interesting works here will be extensively applied and largely cited, and further be written into relevant books because of their generalities and being able to be extensively applied.

Specially, we want to stress that the achieved theory in this paper can be utilized to calculate the general QCD strong interactions and give the precise predictions, further the achieved predictions in the calculations can be exactly measured by current particle physics experiments due to their gauge invariant properties etc, because any physical quantity (relative to gauge transformations) without gauge invariant property cannot be exactly measured [6].

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- [1] C. N. Yang and R. Mills, Phys. Rev. 96 (1): 191.
 - [2] Thomas D. Cohen, Brian Mc Peak, Bendeguz Offertaler, Phys. Rev. C 92, 015204 (2015).
 - [3] E. Ageev et al. (COMPASS Collaboration), Physics Letters B 612, 154 (2005).
 - [4] V. Alexakhin et al. (COMPASS Collaboration), Physics Letters B 647, 8 (2007).
 - [5] A. Airapetian et al. (HERMES Collaboration), Phys. Rev. D 75, 012007 (2007).
 - [6] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang, and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008).
 - [7] X. Ji, Phys. Rev. Lett. 78, 610 (1997).
 - [8] M. Wakamatsu, Phys. Rev. D81, 114010 (2010), arXiv:1004.0268 [hep-ph].
 - [9] X. Ji, Phys. Rev. Lett. **104**, 039101 (2010); X. Ji, Phys. Rev. Lett. **106**, 259101 (2011) [arXiv:0910.5022 [hep-ph]].
 - [10] X. Ji, J. H. Zhang and Y. Zhao, Phys. Rev. Lett. **111**, 112002 (2013) [arXiv:1304.6708 [hep-ph]].
 - [11] Y. Hatta, X. Ji and Y. Zhao, Phys. Rev. D **89**, no. 8, 085030 (2014) [arXiv:1310.4263 [hep-ph]]; X. Ji, J. H. Zhang and Y. Zhao, Phys. Lett. B **743**, 180 (2015) [arXiv:1409.6329 [hep-ph]].
 - [12] B. H. Zhou and Y. C. Huang, Phys. Rev. D84, 047701 (2011); Phys. Rev. A84, 032505 (2011) .
 - [13] C. Huang, Y. C. Huang and B. H. Zhou, Phys. Rev. D 92, 056003 (2015).
 - [14] F. Wang, X. S. Chen, X. F. Lu, W. M. Sun, and T. Goldman, (2009), arXiv:0909.0798 [hep-ph].
 - [15] S. Lerma H., B. Errea, J. Dukelsky et al, Phys. Rev. Lett., 99, 032501 (2007); Sz. Borsanyi, S. Dürr, Z. Fodor et al, Phys. Rev. Lett., 111, 252001 (2013).
 - [16] Review of Particle Physics, Phys. Rev. D 86 (2012) 010001.
 - [17] W. Greiner and J. Reinhardt, Field Quantization (Beijing World Publishing Corp., 2003).
 - [18] M. E. Peskin and D. V. Schroeder, An Introduction to Quantum Field Theory (Beijing World Publishing Corp., 2006).